A Discussion of Fracture Compliance

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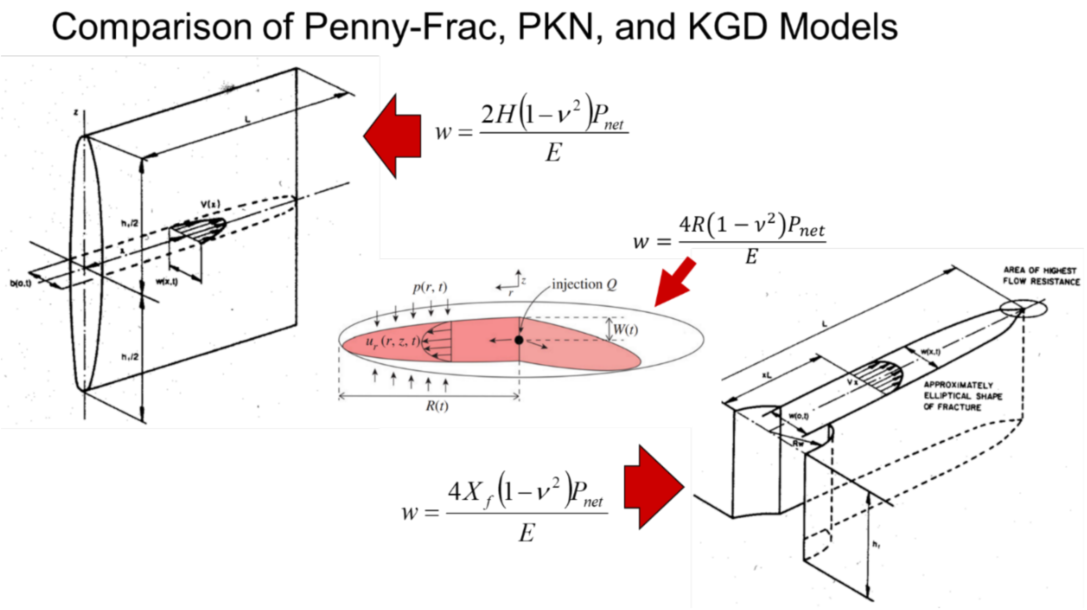
The concept of fracture compliance was introduced primarily by Ken Nolte, as a way to describe how the width of a simple fracture representation might change as the internal pressure, or "net pressure" changes. The model he used was extremely simplified and assumed that fractures have constant-height throughout the length and time-dependent growth of the fracture. These simple models were developed in the late 1950s and early 1960s, when numerical simulation capabilities did not practically exist. With the limited computational processes available, the models were forced to make extremely simplifying assumptions that have been clearly shown to be incorrect. In order to derive fundamental mathematical relations between pressure, time, and geometry Nolte based all his theoretical developments on the "PKN" model.



Comparison of Penny-Frac, PKN, and KGD Models

$$w = \frac{2H\left(1-\nu^2\right)P_{net}}{E}$$

$$w = \frac{4R\left(1-\nu^2\right)P_{net}}{E}$$

$$w = \frac{4X_f\left(1-\nu^2\right)P_{net}}{E}$$

The two commonly used constant height frac models are the PKN (Perkins-Kern-Nordgren) and KGD (Khristianovich and Zheltov, Geertsma and De Klerk) formulations. Both are based on the Sneddon plane-strain linear crack equation for width (1945). The elliptical cross-section of the fracture is assumed to be in the vertical plane in PKN and in the horizontal plane for KGD. This simple difference in assumption causes completely different predictions by the two models. In the PKN model the pressure must rise during pumping to increase width. In the KGD model the width increases with frac length, even with declining pressure. Both assume that width is a function of rate and viscosity, but only because of the relationship between the characteristic frac dimensions and compliance, along with the assumption of an impossible infinite-stress singularity at the fracture tip. The fundamental difference between the model assumptions is that KGD fracs are contained by shear slip at the bounding beds. The PKN model does not allow shear and the frac is contained only by stress contrast but ignores the stress singularity at the vertical fracture tips.

Originally, people thought all fractures were horizontal or parallel to rock bedding. The first application of Sneddon's equation described a radially symmetric or penny-shaped fracture, but all the models are limited by the same simple assumptions. For the PKN and KGD models, which are the primary focus of this discussion, the critical assumption is that rock elastic Young's Modulus remains constant always, and that the characteristic length of the ellipse in Sneddon's Equation (H for PKN and L or Xf for KGD) control the fracture compliance. In the PKN geometry, Nolte define fracture compliance as H/E. For the KGD geometry, compliance is Xf/E, imposing a 2x difference in the purported value. In both the PKN and KGD models, the relationship between pressure and width is controlled by the fracture compliance.

Possibly one of the most severe mistakes ever made, and published, by Nolte is the concept of "fracture height recession" as the mechanism causing what we now refer to as variable storage in a diagnostic fracture injection test (DFIT). His reasoning for this error is understandable when limited by the assumptions of the PKN model. In a variable storage leakoff, the observed pressure after the end of injection tends to decline very slowly, then the pressure decline rate increases through "fracture closure" (also a misnomer that really indicates relaxation of induced strain), then the pressure decline rate decreases as the reservoir transient dominates after "closure" falloff.

Nolte reasoned that the early shallow pressure-time slope indicated a high fracture compliance or easily deformable fracture where a change in fracture volume would not cause a large change in pressure. As the fracture began to "close" or relax, he explained the increasing pressure-time slope to an increase in fracture stiffness or decreasing compliance. In the limited PKN model, with constant modulus and no allowance for shear, and constant isotropic system permeability, the only parameter he could change to vary compliance is H. To explain the change in the pressure-time slope he called the process "fracture height recession". This brings up a fundamental question: If fracture compliance represents the stiffness of a given fracture geometry, do the fracture walls grow back together as the fracture relaxes? Does broken rock regain a resistance to displacement after it relaxes?

Conventional linear-elastic "fracture models" (LEFM), like PKN and KGD, assume the deformed rock faces of a fracture bend like a bow, loaded beam, or flat spring. The figure below shows what the term "fracture compliance" means when applied to the analogy of a PKN/KGD fracture face and a linear spring element. The stiffness or spring constant of the mechanical spring represents the Young's Modulus of the rock, and the loaded length of the spring represents fracture height. For a real fracture to behave in this way there can be no instance of shear anywhere in the fracture face or surrounding rock volume. Incidentally, this is another assumption of the simple models, that the entire rock mass behaves as a homogeneous linear-elastic medium with no possibility of shear planes, interfaces, or induced shear (Mode II or Mode III) fractures.

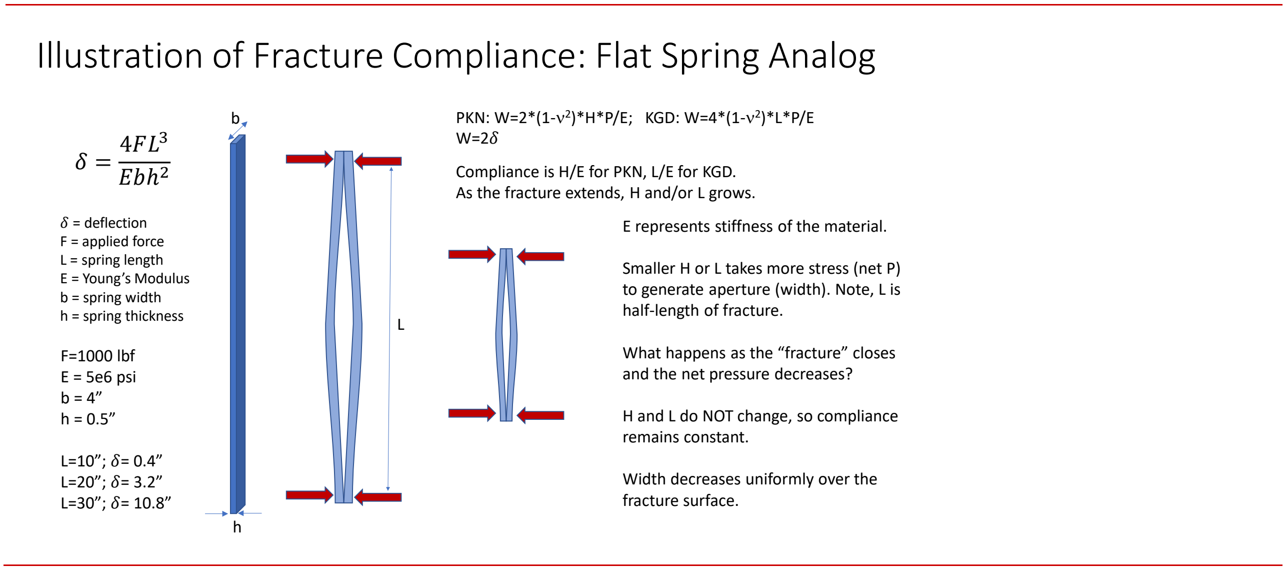


Illustration of Fracture Compliance: Flat Spring Analog

$$\delta = \frac{4FL^3}{Ebh^2}$$

δ = deflection
F = applied force
L = spring length
E = Young's Modulus
b = spring width
h = spring thickness

F=1000 lbf
E = 5e6 psi
b = 4"
h = 0.5"

L=10"; δ= 0.4"
L=20"; δ= 3.2"
L=30"; δ= 10.8"

PKN: $W=2*(1-\nu^2)*H*P/E$; KGD: $W=4*(1-\nu^2)*L*P/E$
$W=2\delta$

Compliance is H/E for PKN, L/E for KGD.
As the fracture extends, H and/or L grows.

E represents stiffness of the material.

Smaller H or L takes more stress (net P) to generate aperture (width). Note, L is half-length of fracture.

What happens as the "fracture" closes and the net pressure decreases?

H and L do NOT change, so compliance remains constant.

Width decreases uniformly over the fracture surface.

The concept of fracture height recession, and a relatively new and equally incorrect theory called "variable compliance" both require the assumption that the fracture mechanically closes starting at the fracture tip (height or length) and that closure proceeds back toward the wellbore or fracture initiation site. In order for the fracture compliance to change, fracture closure must involve the rock regaining mechanical strength to resist fracture opening. The "variable compliance" theory simply applies the same incorrect assumptions that affected Nolte's analysis to what is effectively a KGD model, with height recession replaced by fracture tip recession.

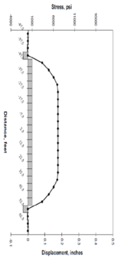
A very simple visual analogy may help to clarify the difference between fracture relaxation and mechanical closure and regrowth of rock integrity. Picture a wooden board or plank. Drive a wedge into the end of the board so that a split (fracture) is generated over some length. Withdraw the wedge. Is the board still broken over the length of the induced split/crack/fracture? Has the wood grown back together? The most important part of this analogy, which applies directly the real hydraulic fracture case is: Does it take the same amount of energy to reopen the existing crack? If the wedge is reinserted, does the entire previous crack open, or must it be generated from the start? I think the answer should be clear.

In the case of a hydraulic fracture, it is important to note that the effective permeability of a crack (not plugged with fines) can be calculated from the square of the crack or slot aperture. A crack width of 0.001" has an equivalent permeability of 54 darcies. That suggests that fluid injected into a relaxed fracture will probably apply an almost uniform pressure over the entire previously created surface area of the fracture. In this case, the entire fracture should reopen along its entire length and height and does not need to be propagated from the point of injection. This argument suggests that the compliance of the fracture on re-opening should be similar to the compliance during relaxation, except for hysteresis in the rock.

As an aside comment, real rocks are not linearly-elastic. When deformed past their elastic limit a residual plastic deformation will remain when the surface load is removed. In terms of fracture relaxation, or "closure", that means that real fracture will always have some small aperture when the deformed rock has relaxed to a neutral stress state. How do we know that the rock has been deformed past it's elastic limit? The occurrence of micro-seismic events indicates shear failure in the rock surrounding the induce hydraulically driven fracture. Shear failure, while not allowed by PKN and KGD models, is an absolute indication of inelastic deformation. When the rock is deformed past its elastic limit, another result is that Young's Modulus changes (decreases) during compaction, and the relaxation modulus differs from the compaction modulus (increases) during relaxation or "closure".

Some highly technical and credentialed researchers have conducted detailed experiments to show that an induced hydraulically-driven fracture relaxes over its entire length during “closure”. The material they used comes as close to a constant modulus, linear-elastic, homogeneous medium as can be obtained. Fracture aperture was accurately measured over the entire surface of the fracture during injection and relaxation. An illustration of their experiment is shown in the figure below. The small inset figure at left shows a simulated fracture aperture profile from a 1998 GOHFER model that closely resembles the observed fracture contour. The detailed study shows that fractures do not close from the tip(s) and compliance, related to the characteristic fracture dimension, does not change during closure. The critical observation from the experiments highlighted in the figure below is that “the crack radius remains constant during the closure process”

Experimental evidence of uniform fracture closure, not “height recession” or “variable compliance”



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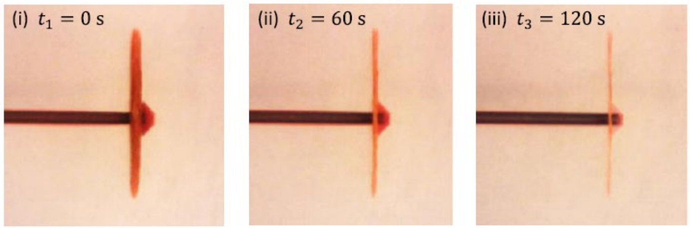

Elastic Relaxation of Fluid-Driven Cracks and the Resulting Backflow

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* Not a bunch of frac-hacks

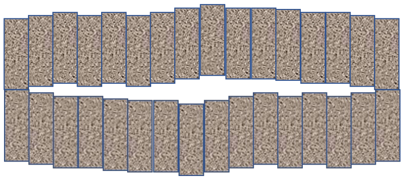
“In our experiments we observed that the crack radius remains a constant during the closure process.” *Physical Review Letters, 117, 268001, Dec 2016*

Any real hydraulic fracturing process in rock generates micro-seismic events, which are associated with shear failures in the rock around the fracture. Shear planes allow energy dissipation and prevent the walls of the fracture from remaining elastically coupled. On a large scale, approaching realistic physical fracture dimensions, each element of the fracture surface is more likely to move independently rather than the fracture surface bending like a bow or spring. In this realization, fracture compliance practically has no meaning. The term compliance was developed by Nolte as a simplified way of estimating the energy forcing fluid out of a distended fracture during relaxation. In the limited PKN model he used, and with the incorrect assumption of constant modulus, the only variable available in the model which could change the expulsion force was H. The fact that H/E in the PKN model, or Xf/E in KGD, represents some estimate of compliance only applies when the fracture is growing and when the deformed rock volume acts as a perfectly linear-elastic medium with no shear. The same assumptions do not apply during relaxation of an existing fracture volume in a shear-decoupled, and more realistic, rock mass.

One more thought about fracture compliance:

What if the rock is not fully elastically coupled?
What if there are shear fractures, bedding planes, weak joints, and natural fractures?
What if each element of the fracture surface is displaced independently by local stress balance?
What if this happens in both vertical and horizontal sections?

In that case, fracture compliance becomes a meaningless academic theory that does not apply to real-world hydraulic fractures in heterogeneous media.
The fracture can close irregularly, with some high-stress or high-modulus layers closing quickly and forming “pinch points”.

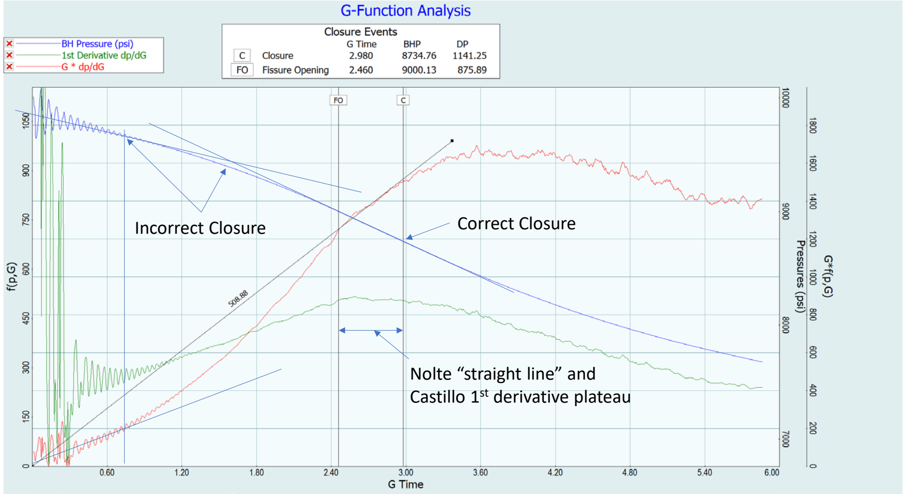


Every “cell” moves independently in a “shear-decoupled” system, not together like a spring or bow.

What we observe as fracture “net pressure” is a complex volume integrated average of the interaction between fluid pressure and rock stress at each element of the fracture surface and surrounding stress-affected rock. Stress and mechanical properties in the rock can change over a distance of centimeters and the “net pressure”, which is generally used to describe the difference between fluid pressure in the fracture and the closure stress in the rock it opposes, is different at every element of the fracture surface. Only the integrated average has meaning, and it is difficult to determine since pressure inside the propagating fracture cannot be measured and the earth stress at each point changes during the fracturing process due to induced strain and pore pressure variations driven by fluid loss and poroelastic compaction (Skempton’s Coefficient).

The figure below shows the correct interpretation of the fracture closure stress from established and proven analysis methods, and two recently proposed and completely inaccurate “closure” picks. The recently proposed “variable compliance” method is based on the incorrect interpretation of changing fracture geometry during relaxation, and that “closure” means the fracture walls are in contact. In Nolte’s perfectly linear-elastic model, the fracture aperture is zero when the fluid pressure in the fracture equals the total stress normal to the fracture face (also not constant in reality). This allowed him to carry out material-balance solutions for “closure” or more accurately “relaxation” time. Real rocks, exposed to strain past their yield point, where they generate micro-seismic emanations, are not elastic and will retain some residual deformation after relaxation of applied net stress.

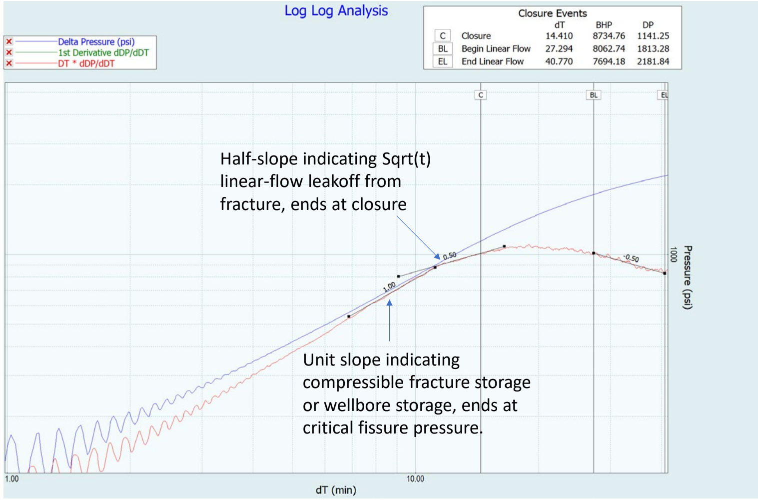
G-Function for Variable Storage Falloff



The deviation from the semi-log derivative straight line, shown in the figure as the separation between the red line and black straight line through the origin, shows up as a unit-slope derivative in the log-log plot of bottomhole pressure change versus shut-in time. This is equivalent to “wellbore storage” in a conventional pressure buildup analysis. In this case the “storage” is caused by changes in fracture volume or pressure recharge from a dilated secondary fracture system affected by the test. Interpretation of real pressure falloff signatures cannot be conducted while assuming the created fracture is planar, or in a homogeneous linear-elastic medium. Any deviation from a straight-line semi-log derivative (red line in the plot) in the G-function analysis indicates some non-planar fracture complexity.

The straight-line portion of the semi-log derivative in the above plot corresponds to a period of fracture relaxation where the recharge or energy input from the complex fracture network has ended, and the fracture relaxation is governed dominantly by leakoff normal to the face of the fracture. This “normal leakoff” can be described by a constant pressure diffusivity solution for one dimensional flow, which predicts the leakoff rate be proportional to the square-root of time. In the log-log plot below, this period of leakoff and relaxation shows up as a ½ slope derivative which ends when the fracture walls reach an equilibrium condition and stop moving. This point represents the minimum in-situ stress normal to the fracture plane, remembering that the fracture is not perfectly planar and that stress is an integrated average and the fracture aperture is not zero.

Closure and Storage on Log-Log Derivative Plot



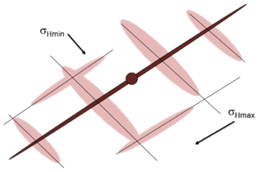
Closure Events		dT	BHP	DP
C	Closure	14.410	8734.76	1141.25
BL	Begin Linear Flow	27.294	8062.74	1813.28
EL	End Linear Flow	40.770	7694.18	2181.84

Half-slope indicating Sqrt(t) linear-flow leakoff from fracture, ends at closure

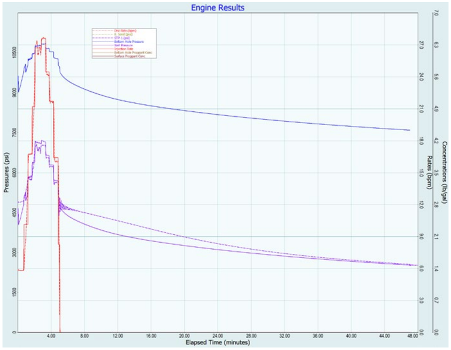
Unit slope indicating compressible fracture storage or wellbore storage, ends at critical fissure pressure.

Assuming that the non-linear derivative signature shown in the example presented here is caused by storage and recharge from a dilated secondary fracture system, its effect on the pressure decline can be modeled. The plot at left, below, shows the simulated pressure decline for a constant compliance planar fracture (solid line), with no secondary storage or recharge, compared to the actual pressure decline for a more complex fracture system (dashed line). The plot at right shows the simulated pressure decline, matching the actual decline, when a slow and declining recharge rate is included in the fracture relaxation modeling. The recharge is driven by the quasi-elastic deformation of the secondary fractures opening against a higher closure stress than the minimum in the earth tensor. The higher stress forcing these fractures closed causes fluid to be displaced from them, into the lower-stress primary fracture, slowing the rate of pressure deecline. Injection, or recharge, slows and stops as the secondary fractures relax. Final “closure” is reached when the primary fracture reaches equilibrium with the minimum stress in the earth tensor.

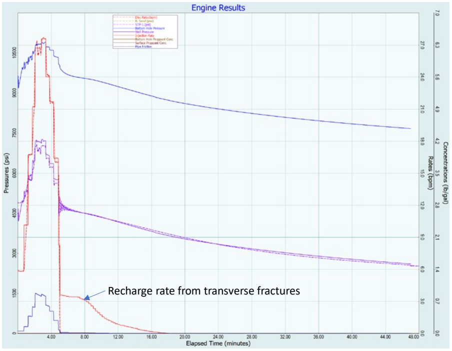
Accounting for Recharge from Transverse Storage



No recharge from storage: Hard shut-in with no return rate

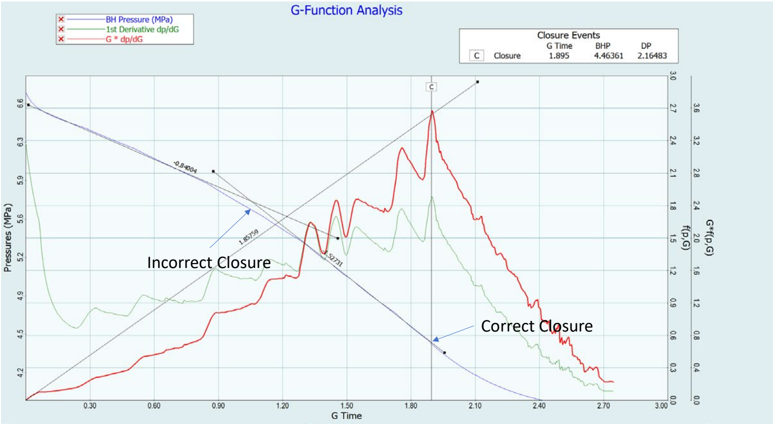


With recharge from storage: Variable return rate



Another fallacious pressure decline analysis that seems to be related to the “variable compliance” method has recently been published to analyze the pressure decay during a pump-in/flow-back test. These tests have been analyzed using the conventional G-function method and the flow-back appears to generate a signature similar to the recharge mechanism. During flow-back, the relaxing fracture walls maintain pressure on the fluid as it is withdrawn from the fracture, causing an apparent “storage” effect. The “new” (and incorrect) method recommends picking the intersection of two tangents to early and late pressure-time linear trends as “closure”. This method is clearly incorrect and lacks any physical basis. Nolte, in his keynote address at the first Hydraulic Fracturing Conference, called this kind of method “picking bumps and wiggles” to select “closure”. The correct minimum in-situ stress can be determined from the peak of the semi-log derivative (only for flowback tests). This “closure pick” also corresponds to the end of the “correct” straight line on the pressure versus G-time curve, as originally proposed by Nolte in 1979.

G-Function Example for Constant-Rate Flowback



CLOSING

In summary, we must note that the industry teems with ambiguous, incorrect, and misused terminology. Fracture “closure” is not closure and does not represent a zero-aperture fracture. “Compliance” is not directly related to H/E or Xf/E and represents an integrated pressure response to a complex fracture change. As noted in SPE 209170-MS, “ISIP” is not the instantaneous shut-in pressure (which has no meaning). Instead, fracture extension pressure (FEP) should be correctly determined and reported. “Net pressure” is often taken as ISIP-Closure, when both are incorrectly determined or reported and can lead to serious misunderstandings of fracture geometry, as does the fracture extension gradient when BH-ISIP/TVD is used. “Net pressure“ cannot be defined as a single value as it varies continuously over the entire fracture with both time and location. Also, SPE 209125-MS explains that “tip screenouts” do not, and have never happened, but the concept and term is still used to incorrectly design fracture treatments.

Misusing terminology and grossly oversimplifying a process as complex as hydraulic fracturing may have been acceptable 60 years ago, when the PKN model was proposed, but no longer. With the availability of more complex diagnostic tools, the industry must be responsible enough to describe and name processes more correctly and to finally retire obsolete ideas and models. As George E. P. Box famously said: “All models are wrong but some are useful”. While the current models (specifically GOHFER) can help us understand fracturing processes more clearly and help make better design decisions, they clearly do not embody all the complexities of the actual treatment. Using grossly oversimplified and incorrect models is not useful and leads to bad designs, decisions, and interpretations.

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